1. (8 marks)

Evaluate the following

(a) $\int (2x+4)^6 \, dx$ (2)

(b) $\int_{\pi/4}^{\pi/2} \left(2\sin(x) - \cos(x)\right) dx$ (4)

(c) $\int \left(x^4 + e^{2x} + \frac{2}{x}\right) dx$ (2)

(b) (i) Given $g(x)=\sqrt{sin(x)}$

show that $g'(x)=\dfrac{cos(x)}{2\sqrt{sin(x)}}$. (2)

(ii) Hence determine $\int -\dfrac{3cos(x)}{\sqrt{sin(x)}}dx$ (2)

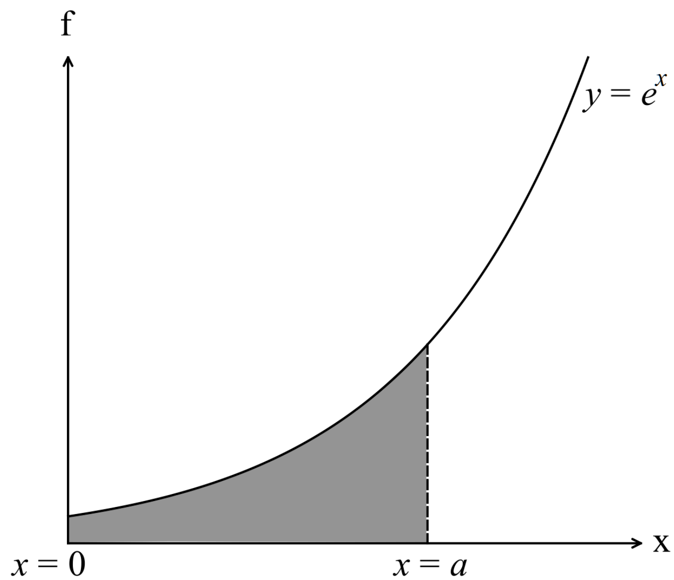
(c) Find $2\int_0^4 (1-f(x))\,dx$ given $\int_0^{10} f(x)\,dx = -6.4$ and $\int_4^{10} f(x)\,dx = 2.3$. (2)

(d) Given $r=\sqrt{t}, \quad t=4x, \quad x=cos(\theta)$

find an expression for $\dfrac{dr}{d\theta}$ as a function of θ. (3)

6. (7 marks)

Consider the diagram below.



(a) Show that the area under the curve $y = e^x$ between $x = 0$ and $x = a$ is $A = e^a - 1.$ (3)

(b) Use a calculus method to determine the increase in area as a increases from 3 to 3.1 units. (4)

7. (6 marks)

A particle with a velocity of $v = 10t - 1 ms^{-1}$.

Determine

(a) an expression for the acceleration and the displacement given the initial displacement is 3 m. (3)

(b) when the particle changes direction. (1)

(c) the distance travelled during the first five seconds. (2)

8. (7 marks)

(a) (i) Find $f'(t)$ given $f(t)=\sqrt{sin(\pi t)}$. (3)

(ii) Hence find $\int_{1/6}^{1/2} \frac{\pi cos(\pi t)}{2\sqrt{sin(\pi t)}} dt$ (2)

(b) Let $F(x)=\frac{d}{dx}\int_1^x \left(\frac{1}{t}\right)dt$

Hence show that $\int_1^2 F(x)\,dx = ln(2)$ (2)

SECTION ONE

1. (8 marks)

(a) $$\int (2x+4)^6\,dx = \frac{(2x+4)^7}{7\times 2} + c = \frac{(2x+4)^7}{14} + c$$ ✓ ✓

(b) $$\int_{\pi/4}^{\pi/2} 2\,sin(x) - cos(x)\,dx$$

$$= \left[-2\,cos(x) - sin(x)\right]_{\pi/4}^{\pi/2}$$ ✓ ✓

$$= -\left(\left(2\,cos\left(\frac{\pi}{2}\right) + sin\left(\frac{\pi}{2}\right)\right) - \left(2\,cos\left(\frac{\pi}{4}\right) + sin\left(\frac{\pi}{4}\right)\right)\right)$$ ✓

$$= -1 + \frac{3}{\sqrt{2}}$$ ✓ $$= -1 + \frac{3\sqrt{2}}{2}$$

(c) $$\int \left(x^4 + e^{2x} + \frac{2}{x}\right)dx = \frac{x^5}{5} + \frac{e^{2x}}{2} + 2\,ln(x) + c$$ ✓✓ -1/error

2. (16 marks)

(a) (i) $$f(x) = ln\left(\frac{x^2-3}{1+x}\right) = ln\left(x^2-3\right) - ln(1+x)$$

$$f'(x) = \frac{2x}{\left(x^2-3\right)} - \frac{1}{(1+x)}$$ ✓ ✓

(ii) $$g(x) = \frac{e^{sin(x)}}{cos(x)}$$

$$g'(x) = \frac{\left(e^{sin(x)}\,cos(x)\right)cos(x) - \left(-sin(x)\right)e^{sin(x)}}{\left(cos(x)\right)^2}$$ ✓ ✓

$$g'(x) = \frac{e^{sin(x)}\left(cos^2(x) + sin(x)\right)}{cos^2(x)}$$ ✓

(iii) $$h(x) = e^x \times ln\left(x^2\right) = 2e^x \times ln(x)$$

$$h'(x) = 2\left(e^x \times ln(x) + \frac{e^x}{x}\right)$$ ✓✓

$$h'(x) = 2e^x\left(ln(x) + \frac{1}{x}\right)$$

(b) (i) Given $g(x)=\sqrt{sin(x)}$ show that $g'(x)=\dfrac{cos(x)}{2\sqrt{sin(x)}}$.

$$g'(x)=\frac{1}{2}(sin(x))^{\frac{-1}{2}}\times cos(x) \quad ✓ \quad ✓$$

$$g'(x)=\frac{cos(x)}{2\sqrt{sin(x)}}$$

(ii) $$\int -\frac{3cos(x)}{\sqrt{sin(x)}}dx=-3\times 2\int\frac{cos(x)}{2\sqrt{sin(x)}}dx \quad ✓$$

$$=-6\int\frac{cos(x)}{2\sqrt{sin(x)}}dx$$

$$=-6\sqrt{sin(x)}+c \quad ✓$$

(c) Given $\int_0^{10} f(x)\,dx=-6.4$ and $\int_4^{10} f(x)\,dx=2.3$.

$$2\int_0^4 \; = 2\int_0^{10} \; - 2\int_4^{10} \quad ✓$$

$$2\int_0^4 (1-f(x))dx = 2\left(\int_0^4 1dx-\int_0^4 f(x)dx\right)$$

$$=2\left(\left[x\right]_0^4-\left(\int_0^{10} f(x)dx-\int_4^{10} f(x)dx\right)\right)$$

$$=2(4-(-6.4-2.3))$$

$$=2(4+8.7)$$

$$=2\times 12.7$$

$$=25.4 \quad ✓$$

(d) $$\frac{dr}{d\theta}=\frac{dr}{dt}\times\frac{dt}{dx}\times\frac{dx}{d\theta} \qquad t=4x=4cos(\theta) \quad ✓$$

$$\frac{dr}{d\theta}=\frac{1}{2}t^{-1/2}\times 4\times(-sin(\theta)) \quad ✓$$

$$\frac{dr}{d\theta}=-\frac{2\,sin(\theta)}{\sqrt{t}}$$

$$\frac{dr}{d\theta}=-\frac{2\,sin(\theta)}{\sqrt{4cos(\theta)}}$$

$$\frac{dr}{d\theta}=-\frac{sin(\theta)}{\sqrt{cos(\theta)}} \quad ✓$$

SECTION TWO

6. (7 marks)

(a) $A=\int_0^a e^x dx=\left[e^x\right]_0^a=e^a-e^0=e^a-1$ ✓ ✓ ✓

(b) $A=e^a-1$

$\dfrac{dA}{da}=e^a$ ✓

$\dfrac{\delta A}{\delta a}\approx\dfrac{dA}{da}$

$\delta A\approx\dfrac{dA}{da}\times\delta a$ ✓

$At\quad a=3,\ \delta a=0.1$ ✓

$\delta A\approx e^3\times 0.1$

$\delta A\approx 2.0086$ ✓

7. (6 marks)

(a) $v=10t-1ms^{-1}.$

$a=10\ ms^{-2}$ ✓

$x=\int(10t-1)\,dt$

$x=5t^2-t+c$ ✓

$At\ t=0, x=3$

$x=5t^2-t+3$ ✓

(b) Changes direction when $v=0$ i.e. at t = 0.1 ✓

(c) $At\ t=0,\ x=3$

$At\ t=0.1,\ x=0.05-\ 0.1+3=\ 2.95$ ✓

$At\ t=5,\ x=123$

Distance travelled $=123-2.95+0.05=120.1\ m$ ✓

8. (7 marks)

(a) (i) $f(t)=\sqrt{sin(\pi t)}$

$f'(t)=\frac{1}{2}(sin(\pi t))^{\frac{-1}{2}}\pi cos(\pi t)$ ✓ ✓

$f'(t)=\frac{\pi cos(\pi t)}{2\sqrt{sin(\pi t)}}$ ✓

(ii) $\int_{1/6}^{1/2}\frac{\pi cos(\pi t)}{2\sqrt{sin(\pi t)}}dt=\left[\sqrt{sin(\pi t)}\right]_{1/6}^{1/2}$ ✓

$=\sqrt{sin\left(\frac{\pi}{2}\right)}-\sqrt{sin\left(\frac{\pi}{6}\right)}$

$=1-\frac{1}{\sqrt{2}}$ ✓

(b) $F(x)=\frac{d}{dx}\int_1^x\left(\frac{1}{t}\right)dt=\frac{1}{x}$ ✓

$\int_1^2 F(x)dx=\int_1^2\frac{1}{x}dx=\left[ln(x)\right]_1^2=ln(2)-ln(1)=ln(2)$ ✓

9. (8 marks)

(a) Turning points occur when $f'(x)=0.$ ✓

There are no points where this occurs so there are no turning points. ✓

Likewise, there are no points where $f''(x)=0,$ so there are no points of inflection. ✓

(b) $y=f''(x)<0$ which suggests that the concavity is concave downwards for all x values. ✓✓

(c) $f(x)=ln(x),\quad f'(x)=\frac{1}{x},\quad f''(x)=\frac{-1}{x^2}$ ✓ ✓ ✓

12. (7 marks)

(a) Area $= \int_{0.05}^{1.47} \left(ln(x) - (e^x - 4) \right) = 1.68$ units^2 ✓ ✓ ✓

(b) (i) $P(0) = 22\,ln(3) = 24.169 \approx 24$ ✓ ✓

(ii) $100 = 22\,ln(t+3)$

$t = 91.203$ ✓

$2002 + 91 = 2093$

The population will reach 100 in 2094 or just into 2094. ✓

13. (7 marks)

(a) $A = x \times y$ ✓ $y^2 = 1 - x^2$

$A = x\sqrt{1-x^2}$ ✓

(b) Maximum area when $\dfrac{dA}{dx} = 0$ and $\dfrac{d^2A}{dx^2} < 0$ ✓

$$\frac{dA}{dx} = -\frac{2x^2 - 1}{\sqrt{1-x^2}}$$ ✓

$$\frac{d^2A}{dx^2} = -\frac{2x^3\sqrt{1-x^2} + 4x\sqrt{\left(1-x^2\right)^3} - x\sqrt{1-x^2}}{\left(x^2-1\right)^2}$$ ✓

$If\ \dfrac{dA}{dx} = 0, \quad 2x^2 - 1 = 0$

$x^2 = 0.5$

$x = \dfrac{1}{\sqrt{2}} \quad x > 0$

Max or min?

$$\frac{d^2A}{dx^2} = -\frac{4\left(\frac{1}{2\sqrt{2}}\right)\sqrt{0.5} + 4\left(\frac{1}{2\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\left(\sqrt{0.5}\right)}{2} = -\frac{\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\sqrt{0.5} + 4\left(\frac{1}{2\sqrt{2}}\right)}{2} < 0$$ ✓

∴ max

$x = \dfrac{1}{\sqrt{2}}, \qquad y = \sqrt{1 - \left(\dfrac{1}{\sqrt{2}}\right)^2} = \dfrac{1}{\sqrt{2}}$

Therefore $P\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ ✓